x^2+x+1

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{(x+h)^2+(x+h)+1-(x^2+x+1)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{+x}+h\cancel{+1}\cancel{-x^2}\cancel{-x}\cancel{-1}}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}=\lim_{h\to 0}\frac{\cancel{h}(2x+h+1)}{\cancel{h}}$$

$$=\lim_{h\to 0}(2x+h+1)=2x+1$$

Section 3.2 - Differentiability

A function will not have a derivative at a point P(a, f(a)) where the slopes of the secant lines,

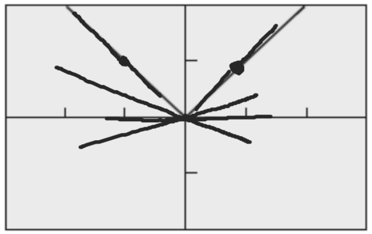
$$\frac{f(x)-f(a)}{x-a},$$

fail to approach a limit as x approaches a.

4 situations...

Example

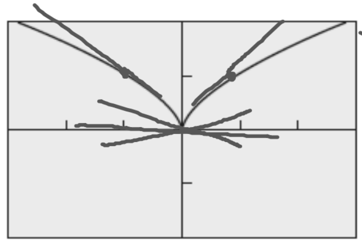
corner

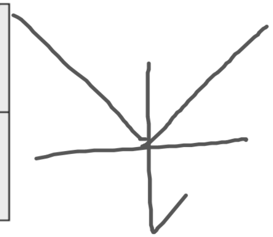


[–3, 3] by [–2, 2]

1.) A corner, where the one-sided derivatives differ : Example, $f(x) = |x|$

2. Cusp - where the slopes of the secant lines approach ∞ from one side and $-\infty$ from the other



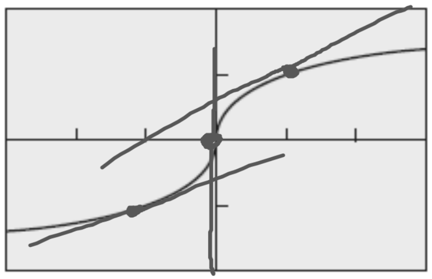


[–3, 3] by [–2, 2]

example $y = x^{2/3}$

3. A Vertical Tangent - where the slopes of the secant lines approach either ∞ or -∞ from both sides

There is a vertical tangent line at x = 0

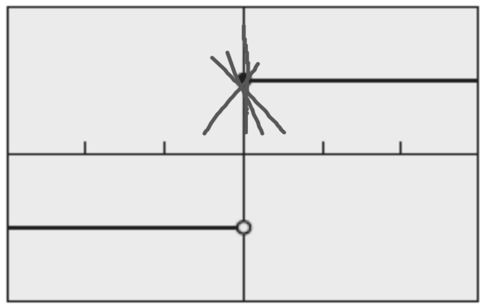


[–3, 3] by [–2, 2]

example $y = \sqrt[3]{x}$

4.) A discontinuity - which will cause one or both of the one-sided derivatives to be non-existent.

There is a discontinuity at x = 0

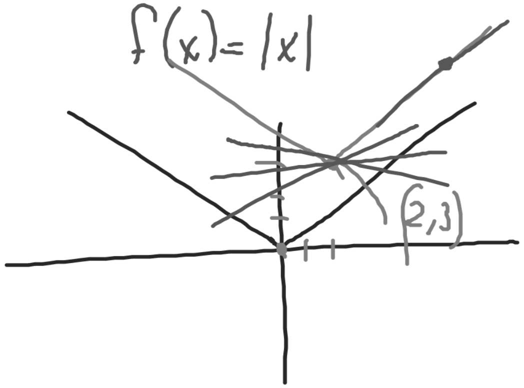


[–3, 3] by [–2, 2]

Example

Find all points in the domain of $f(x) = |x - 2| + 3$ where f is not differentiable.

Right 2 up 3



Differentiability implies local linearity

- when you zoom in really really close to a point of tangency, the function looks more and more like the tangent line at that point!

    - differentiable curves will straighten out when we zoom in on them at a point of differentiability

If a function is differentiable at a point, it is locally linear!!!

Slope from left MUST equal the slope from the right

Tangent lines exist at that point as well

$y = |x|$ at $x = 0$

$$y = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Explain why there is no derivative at x = 0?

$=x$ $f'(x) = \lim_{h \to 0^+} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^+} \frac{x+h-x}{h}$

$= \lim_{h \to 0^+} \frac{h}{h} = 1$

$=-x$ $f'(x) = \lim_{h \to 0^-} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0^-} \frac{-(x+h) - (-x)}{h}$

$= \lim_{h \to 0^-} \frac{-x-h+x}{h} = -1$

Explain why $y = x^{2/3}$ is not differentiable at $x = 0$.

Explain why $y = \sqrt[3]{x}$ is not differentiable at $x = 0$.

NDER f(a)

The numerical derivative of f at a point A.

Example

Computer NDER (x^3, 2), the numerical derivative of x^3 at $x = 2$.

Computer NDER (|x|, 0), the numerical derivative of |x| at x = 0.

Theorem

Differentiability Implies Continuity

If f has a derivative at x = a, then f is continous at x = a.

If a function is discontinous at a point, it is NOT differentiable at that point.

If a function is differentiable at a point, it is continuous at that point.

Theorem

Intermediate Value Theorem for Derivatives

If a and b are any two points in an interval on which f is differentiable, then f ' takes on every value between f '(a) and f '(b).

Example

Does any function have the Unit Step Function as its derivative?

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Show that the function

$$f(x) = \begin{cases} 0, & -1 \leqslant x < 0 \\ 1, & 0 \leqslant x \leqslant 1 \end{cases}$$

is not the derivative of any function on the interval $-1 \leqslant x \leqslant 1$